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MODELLING THE GUARANTEE LIABILITY UNDER UNIT-LINKED CONTRACTS

Empirical studies

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Abstract

Unit-linked contracts are wrapped with some death, maturity and accumulation guarantees such as the guaranteed minimum maturity benefit, the guaranteed minimum death benefit, the guaranteed minimum accumulation benefit, the guaranteed minimum income benefit, and/or the guaranteed minimum surrender benefit. According to Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for modelling liabilities for unit-linked insurance contracts with incorporated investment guarantees. The aim of this study is to offer an optimal theoretical approach for simulating liabilities for unit-linked life insurance contracts with incorporated death benefit and maturity benefit.

Introduction

The scope of life insurance is to provide financial security to policyholders and their families. Traditionally, this security has been provided by means of a lump sum payable contingent on the death or survival of the insured life. The sum insured would be fixed and guaranteed. The policyholder would pay one or more premiums during the term of the contract for the right to the sum insured. Traditional actuarial techniques have focused on the assessment and management of life-contingent risks: mortality and morbidity. The investment side of insurance generally has not been regarded as a source of major risk. This was a reasonable assumption, where guaranteed benefits can be broadly matched or immunized with fixed-interest instruments. But insurance markets around the world are changing. The public has become more aware of investment opportunities outside the insurance sector, particularly in mutual fund type investment media. As a consequence policyholders want to enjoy the benefits of equity investment in conjunction with mortality protection, and insurers around the world have developed equity-linked contracts to meet this challenge. Although some contracts pass most of the asset risk to the policyholder and involve little or no investment risk for the insurer, it was natural for insurers to incorporate payment guarantees in these new innovative contracts (Hardy, 2003).

A unit-linked life insurance policy with an investment guarantee is an insurance policy whose benefit payable on death or at maturity consists of the greater of some guaranteed amount and the value of a reference portfolio which is defined by the deemed investment of a predetermined component of the policy premium in a portfolio of common stocks or mutual fund - the reference fund (Brennan & Schwartz, 1979).

Pricing of unit-linked life insurance contracts has generated much interest among researchers and practitioners in the last two decades (Romanyuk, 2006). The payoff in these types of contracts contains both financial and insurance risk elements (Argesanu, 2004), which have to be priced so that the resulting premium is fair to both the seller (insurer) and the buyer (policyholder) of the contract (Romanyuk, 2006).

Due to the financial instability caused by the Global Crisis and the amplification of market competitiveness (Bojan, Corovei & Trenca, 2014), insurers from international markets have started to incorporate guarantees in unit-linked products. Investment guarantees are very popular features in life insurance policies because in addition to paying a death benefit or a maturity benefit, these policies are tied to the return of an underlying asset or an actively managed portfolio. Thus, the policy also acts as an investment because the investor’s capital is credited with a minimum return. In exchange for this protection, the policyholder pays a higher premium, reflecting the market risk assumed by the insurance company (Augustyniak & Boudreault, 2012).

Unit-linked contracts are wrapped with some death, maturity and accumulation guarantees such as the guaranteed minimum maturity benefit (GMMB), the guaranteed minimum death benefit (GMDB), the guaranteed minimum accumulation benefit (GMAB), the guaranteed minimum income benefit (GMIB), and/or the guaranteed minimum surrender benefit (GMWB) (Gaillardetz, 2006).

The guaranteed minimum maturity benefit (GMMB) guarantees the policyholder a specific monetary sum at the maturity of the contract. This guarantee offers downside protection for the policyholder’s funds, with the upside being participation in the underlying stock index or fund or combination of funds. In general a simple GMMB might be a guaranteed return of insurance premium if the underlying index, fund or combination of funds falls over the term of the insurance (with an upside return of some proportion of the increase in the underlying index, fund or combination of funds in case of a favourable evolution of index, fund or combination of funds performances over the contract term). The guarantee may be fixed or subject to regular or equity-dependent increases.

The guaranteed minimum death benefit (GMDB) guarantees the policyholder a specific monetary amount upon death during the term of the contract. Also the death benefit may simply be the original premium, or may increase at a fixed rate of interest (Hardy, 2003).

According to Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for modelling liabilities for unit-linked insurance contracts with incorporated investment guarantees.

The aim of this study is to offer an optimal theoretical approach for simulating liabilities for unit-linked life insurance contracts with incorporated death benefit and maturity benefit. This study contributes to the existing literature regarding the issue of appropriate method for simulating liabilities for life insurance contracts, with an exclusive focus on the unit-linked life insurance contracts with investment guarantees.

The structure of this paper is as follows: Section 1 discusses some previous research on the issue. Section 2 outlines the methodology. The empirical results regarding the simulation of guarantees liabilities under unit-linked contracts are presented in Section 3. Section 4 provides a summary of the main findings and some concluding remarks.

Literature review

There is an extensive literature on the pricing, hedging and risk management of these contracts. See for example, Boyle and Schwartz (1977), Brennan and Schwartz (1979), Hardy (2003), Argesanu (2004), Gaillardetz (2006), Romanyuk (2006), Reichenstein (2009), Augustyniak and Boudreault (2012), etc. Boyle and Schwartz (1977), and Brennan and Schwartz (1979) were the first articles that elegantly described some of the option elements of life insurance products and demonstrated how the Option Pricing Theory of Black and Scholes could be applied to value these contracts. Hardy (2003) discusses the modelling and risk management for equity-linked life insurance; the focus of his research is on stochastic modelling of embedded guarantees that depend on equity performance. Argesanu (2004) focuses on the risk analysis and hedging of variable annuities in incomplete markets. Romanyuk (2006) describes the problem of appropriate pricing of equity-linked life insurance contracts and hedging of the risks involved, and proposes the use of two types of imperfect hedging techniques: quantile and efficient hedging. Gaillardetz (2006) introduces a pricing method for equity-indexed annuities and valuates these products by pricing its death benefits and survival benefits separately.

Theoretical framework

Risk management of unit-linked insurance requires a full understanding of the nature of the liabilities. In this section, the authors will demonstrate how to use empirical simulation to determine the liability distribution under the guarantee (Hardy, 2003). Under the financial engineering approach, the capital requirement is used to construct a replicating portfolio that will, at least approximately, meet the guarantee when it becomes due. However, this simulation of the liabilities is also important to the financial engineering approach to risk management for the following reasons: there will be transactions costs and the rebalancing of the hedge will be at discrete time intervals rather than continuously. In this case, the assets and liabilities are very closely linked, and the authors need to model both simultaneously. The fund and cash-flow variables are as follows:

- G- represents the guarantee level per unit investment, subscripted G_t if it can change over time.
- F_T- denotes the market value of the separate account at assuming the policy is still fully in force. We assume that the management charge or management expense ratio (MER) is deducted from the fund at the beginning of each year. It is convenient sometimes to distinguish between the fund immediately before these year-end transactions and the fund immediately after. Let F_t- represents the year-end fund at t before these transactions, and let F_t+ represents the year-end fund after the transactions. Where the sign – or + is missing, the authors assume +.
- S_t defines the value of the underlying equity investment at t , where S_0 is assumed for convenience to be equal to 1.
- m denotes the management charge rate deducted from the separate account, per year. The portion available for funding the guarantee cost is m_c, called the margin offset. This may be split by benefit so that, for example, for a joint guaranteed minimum maturity benefit (GMMB) and guaranteed minimum death benefit (GMDB) contract the total risk charge per year would be $m_c = m_m + m_d$, where m_m is allocated to GMMB and m_d is allocated to the GMDB.
- M_t denotes the income at from the guarantee risk charge.
- C_t denotes the liability cash flow at from the contract, net of the income from M_t , allowing for deaths and withdrawals.
- L_0 represents the present value of future liabilities, discounted at a constant risk-free force of interest of r per year.
- tP_x^{τ} is the probability that a life currently aged x dies before age x+t .
- $t|1q_x^d$ is the probability that a policyholder aged years is still in force after t years, but dies in force before the expiry of a specific number of years (in this case after 1 year).

The relationships between these variables, assuming that the margin offset is collected yearly in advance, are:

$$\mathrm{F_t} -= \frac{\mathrm{S_t}}{\mathrm{S_{t-1}}} \mathrm{F_{t-1}} + \qquad (1)$$

$$\mathrm{F_t} += \mathrm{F_t} - \ (1 - \mathrm{m}) = \mathrm{F_{(t-1)}}(1 - \mathrm{m}) \frac{\mathrm{S_t}}{\mathrm{S_{t-1}}} \quad (2)$$

So for the integer t and u, and assuming no cash injections into the fund between t and t+u:

$$F_{(t+u)^+} = F_t \frac{S_{t+u}(1-m)^u}{S_t} \quad (3)$$

Now let F_{0-} be the fund at the valuation date (or at policy issue date, in which case it is the policy single premium), then:

$$F_t = F_{0-} \frac{S_t(1-m)^t}{S_0} \qquad (4)$$

The margin offset income, which is the income allocated to funding the guarantee, is:

$$M_t = (F_{t-})m_c = m_c F_{0-} \frac{(1-m)^t S_t}{S_0} \quad (5)$$

Guaranteed minimum maturity benefit

In this section, the authors show how to generate the distribution of the present value of the guarantee liability for a simple guaranteed minimum maturity benefit policy held by a life aged x with remaining duration n years. It is assumed a yearly discrete time model for equity returns and management charges. Withdrawals and deaths are assumed to occur at year ends (Hardy, 2003).

Since S_t is a stock index, it is assumed that $S_0 = 1.0$ so that S_t is the accumulation factor for the period from time 0 to time t. Also:

$$(G - F_n)^+ = \max(0, G - F_n) \qquad (6)$$

Then:

$$C_t = -tP_x^{\tau}M_t \qquad t = 0,1,\dots,n-1 \quad (7)$$

And:

$$C_n = -nP_x^{\tau}(G - F_n)^+ \qquad (8)$$

Then:

$$L_0 = \sum_{t=0}^{n} C_t e^{-rt} \qquad (9)$$

C_t and L_0 can be calculated for each stock index scenario, and distributions for the cash flows in different years and for the present value random variable can all be simulated.

Guaranteed minimum death benefit

Assume no reset or rollover benefit; the death benefit is the greater of the initial investment and the fund value at death. Using a deterministic approach to the death benefit is equivalent to assuming that tq_x lives per policy die in the interval (0, t). The liability cash flow for the benefit at t is therefore (Hardy, 2003):

$$C_t = -tP_x^{\tau}M_t + t - 1|1q_x^{d}(G - F_t)^+ \qquad t = 0,1,\dots,n \quad (10)$$

$$C_t = -tP_x^{\tau}F_{0-}S_t(1-m)^{t-1}m + t - 1|1q_x^{d}(G - F_{0-}S_t(1-m)^t)^+ \quad (11)$$

M_t^d is the risk charge income in respect of the death benefit.

Numerical simulation

In this research the authors took into consideration a unit-linked life insurance contract which offers a combined GMMB and GMDB. This research implies a contract with a GMMB and a GMDB at a fixed guarantee level, with the following features:

- Let $x = 40\ years$, $F_{0-} = 6000\ LEI$, $G = 6000\ LEI$, $M = 0.05\ per\ year$, $m_c = 0.04\ per\ year$.
- Let the remaining contract term be 8 years.
- The dependent death and withdrawal rates were calculated using the Romanian Institute of Actuaries male annuitants' mortality rates.
- Let BET be the equity index. The data of the study consists of yearly closing values of BET index. BET is the reference index for the Bucharest Stock Exchange (BVB) market. BET is a free float weighted capitalization index of the most liquid 10 companies listed on the BVB regulated market. The index methodology allows BET to be a good underlying for derivatives and structured products. All the closing prices of Bet index are collected from BVB database. The daily data is taken from December 31, 2005 to December 31, 2013.

The result of this particular scenario is given in Table 1. The margin offset is received in advance, so there is no income at the end of the final year .The death benefit under the guarantee is greater than zero only on death in the last 6 years; for the rest of the period the fund is larger than the guarantee. At the end of the contract, the fund is worth slightly less than the guarantee, so a GMMB is due.

At a risk-free annual rate of interest of 6.76 percent per year, the net present value of future liability for this scenario (the sum of the cash flow present values) is -63.33 LEI. The negative sign implies a net income.

Conclusions

Unit-linked life insurance contracts are popular and widely used on the insurance market. Due to the financial instability caused by the Global Crisis and the amplification of market competitiveness, insurers from international markets have started to incorporate guarantees in unit-linked products. They provide either death benefit or maturity benefit or both. The benefits are linked to an underlying asset with or without certain guarantees so that the policyholders have the opportunity to participate in the financial market and (eventually) be protected from the downside development of the financial market (Li & Szimayer, 2011). In recent years insurers have provided more flexible products that combine the death or the maturity benefit coverage with a significant investment element, as a way of competing for policyholders' savings with other financial institutions, for example: banks, open-ended investment companies, stock markets, exchange markets, etc (Dickson, Hardy & Waters, 2009).This paper offers a theoretical framework for modeling guarantee liabilities under unit-linked life insurance contracts. According to the empirical results, at a risk-free annual rate of interest of 6.76 percent per year the net present value of future liability for this scenario is -63.33 LEI, so the negative sign implies a net income.

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Tables

Table 1

GMMB/GMDB liability cash flow projection

Year	***Equity Index S_t***	***F_{t-}***	[illegible]	[illegible]	***Margin Offset Income***	***Death Benefit and Maturity Benefit Outgo***	***C_t***
0	1.0000	6000.00	1.0000	0.0046	240.00		-240.00
1	1.2223	6967.07	0.9954	0.0051	278.68	0.0000	-277.40
2	1.4918	8078.25	0.9904	0.0056	323.13	0.0000	-320.01
3	0.4405	2265.97	0.9848	0.0062	90.64	22.9734	-89.26
4	0.7122	3480.49	0.9787	0.0064	139.22	16.0251	-136.25
5	0.8000	3713.94	0.9723	0.0066	148.56	15.1410	-144.44
6	0.6585	2904.34	0.9657	0.0073	116.17	22.5365	-112.19
7	0.7819	3276.09	0.9584	0.0081	131.04	21.9771	-125.59
8	0.9860	3924.71	0.9503	0.0086	0.00	1972.1872	1972.19

Source: Authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database